Equation (8) is therefore a closed-form coupled twist distribution for a composite wing with the warping effects accounted for, where

$$\bar{y}_0 = y_0/\ell_0 \tag{9}$$

When Eq. (8) is evaluated at the wing tip and compared to an equivalent expression predicted by St. Venant's theory, the following expression is obtained:

$$\frac{\alpha_0(1)}{\alpha_0(1)_{\text{St.V}}} = 1 - \frac{\tanh 4\bar{\lambda}_c}{2\bar{\lambda}_c} - \frac{1}{9\bar{\lambda}_c^2}\left(\frac{1}{\cosh 4\bar{\lambda}_c} - 1\right) \tag{10}$$

where $\alpha_0(1)$ is the wing tip twist given by Eq. (8) while $\alpha_0(1)$ is the wing tip twist given by the St. Venant torsion theory. A plot of Eq. (10) is shown in Fig. 1.

Steady-State Concentrated Tip Twist Loads

If the wing is under the influence of a concentrated twisting moment F_0 at the tip as a result of a steady-state coupled bending torsion displacement, the exact closed-form twist distribution that satisfies these equations of motion and their associated boundary conditions is given by

$$\alpha_0(\bar{y}_0) = \frac{6F_0\ell_0}{c_0^3(4\bar{\lambda}_c)^2}\left[\bar{y}_0 - \frac{\sinh 4\bar{\lambda}_c y_0}{4\bar{\lambda}_c} + \frac{\tanh 4\bar{\lambda}_c}{4\bar{\lambda}_c} \times (\cosh\bar{\lambda}_c\bar{y}_0 - 1)\right] \tag{11}$$

When the twist distribution given by Eq. (11) is evaluated at the wing tip and compared to its counterpart predicted by the St. Venant torsion theory, the following expression is obtained:

$$\frac{\alpha_0(1)}{\alpha_0(1)_{\text{St.V}}} = 1 - \frac{\tanh 4\bar{\lambda}_c}{4\bar{\lambda}_c} \tag{12}$$

It should be noted that the ratio given by Eq. (12) was plotted for the real values of $\bar{\lambda}_c$ in Refs. 5 and 6 and was shown to represent conditions where any errors resulting from using St. Venant's torsion theory are conservative (overdesign rather than underdesign). In this analysis, Eq. (12) is examined when $\bar{\lambda}_c$ is imaginary, which is possible if L_2 is very large. Under such circumstances, Eq. (12) becomes

$$\frac{\alpha_0(1)}{\alpha(1)_{\text{St.V}}} = 1 - \frac{\tan 4\bar{\lambda}_c}{4\bar{\lambda}_c} \tag{13}$$

Figure 2 depicts the conditions given by Eq. (13). It is therefore seen from the figure that there are certain ranges of $\bar{\lambda}_c$ for which nonconservative errors are possible by using the St. Venant twist theory.

Results and Conclusions

The results are shown in Figs. 1 and 2. Figure 1 shows a comparison of the static wing tip twist obtained in the present study and that obtained via St. Venant's twist theory in the presence of statically distributed forces and low-to-moderate coupling. Figure 2 shows the trend for concentrated forces and substantial coupling. In Fig. 1, it is seen that the presence of coupling makes the errors of St. Venant's theory worse. This seems to suggest that the more sophisticated theory is more important for wings with coupling (e.g., wings aeroelastically tailored using elastic cross coupling).

Figure 2 also shows that nonconservative errors $\{|[\alpha_0(1)]/[\alpha_{0\text{St.V}}^{(1)}]|>1\}$ are possible.

Using Figs. 1 and 2, the following conclusions can be summarized: 1) ignoring warping arbitrarily using St. Venant's theory could result in very significant errors (as high as over 80%) in analytical results for composite aircraft wings; 2) warping is more important (St. Venant's theory is less accurate) for wings with coupling; and 3) St. Venant's theory (which has always been shown to be conservative[1,2] can be nonconservative or St. Venant's approximation can lead to an unsafe design error (underdesign rather than overdesign from a stability point of view).

Acknowledgments

Research sponsored by the Air Force Office of Scientific Research (AFSOR), under Contracts F49620-85-C-0090 and F49620-87-C-0046. The United States Government is authorized to reproduce and distribute reprints for governmental purposes notwithstanding any copyright notation hereon.

The author is grateful to the AFOSR for sponsoring this research. He is particularly thankful to Dr. Anthony K. Amos and Dr. A. Nachman, the Program Managers who monitor the research program, for their constructive criticism. He also appreciates various comments from Dr. Sinclair Scala and Mr. Julian Berman, both of Fairchild Republic Company, and Professor Terrence Weisshaar of Purdue University.

References

[1]Reissner, E. and Stein, M., "Torsion and Transverse Bending of Cantilevered Plates," NACA TN-2369, June 1951.

[2]Bisplinghoff, R. L., Ashley, H., and Halfman, R. L., *Aeroelasticity*, Addison-Wesley, Reading, MA, 1955.

[3]Petre, A., Stanescu, C., and Librescu, L., "Aeroelastic Divergence of Multicell Wings (Taking their Fixing Restraints into Account)," *Revue de Mechanique Appliquee*, Vol. 12, No. 6, 1961, pp. 689-698.

[4]Crawley, E. F. and Dungundji, J., "Frequency Determination and Nondimensionalization for Composite Cantilever Plates," *Journal of Sound and Vibration*, Vol. 72, No. 1, 1980, pp. 1-10.

[5]Oyibo, G. A. and Berman, J. H., "Influence of Warpage on Composite Aeroelastic Theories," AIAA Paper 85-0710, April 1985.

[6]Oyibo, G. A. and Berman, J. H., "Anisotropic Wing Aeroelastic Theories with Warping Effects," DGLR Paper 85-57, April 1985.

Investigation of Internal Singularity Methods for Multielement Airfoils

D. R. Chen* and M. J. Sheu†

National Tsing Hua University, Hsinchu, Taiwan, Republic of China

I. Introduction

A NUMBER of integral equation methods are currently used for solving linear potential flows about single airfoils, e.g., Hess and Smith,[1] Bristow,[2] Hess,[3,4] Maskew and Woodward,[5] and Chen and Dalton,[6] and about multielement airfoils, e.g., Banyopadhyay et al.[7] The comparisons of the solutions of these numerical methods for single airfoils are shown in Ref. 8. The analytical solution for two-dimensional potential flow past the airfoils had been presented by Williams[9] and James.[10] The method developed by Garrick[11] can analyze flow about two-element airfoils. The method developed by Ives,[12] Halsey,[13] and Suddhoo and Hall[14] can analyze the flow about multielement airfoils.

A numerical solution, which is obtained by the internal linear-vortex-source flat-element method, is comared with the

Received Nov. 9, 1987; revision received Feb. 23, 1988. Copyright © American Institute of Aeronautics and Astronautics, Inc., 1988. All rights reserved.

*Postgraduate Student, Power Mechanical Engineering Department.

†Associate Professor, Power Mechanical Engineering Department. Member AIAA.

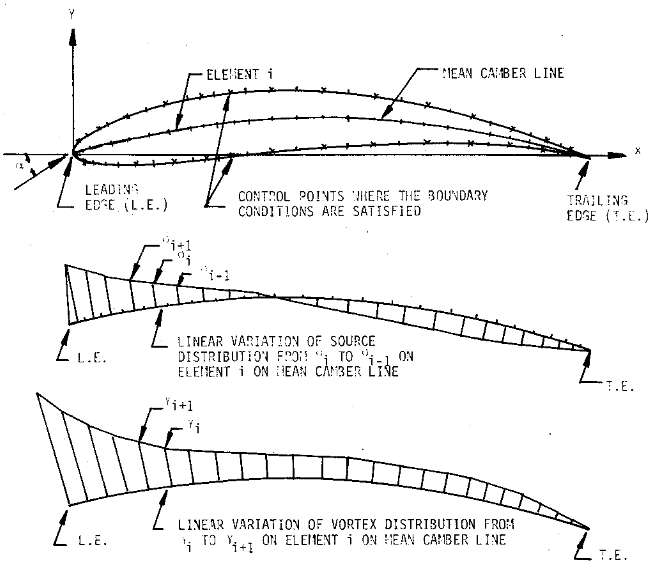

Fig. 1 Distribution of singularities and control points for internal linear vortex-source model.

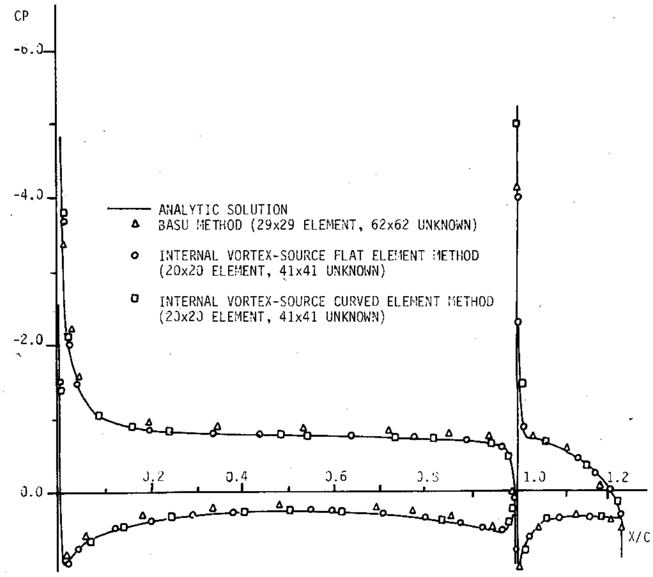

Fig. 2 Pressure distributions of internal singularity method for two-element airfoil.

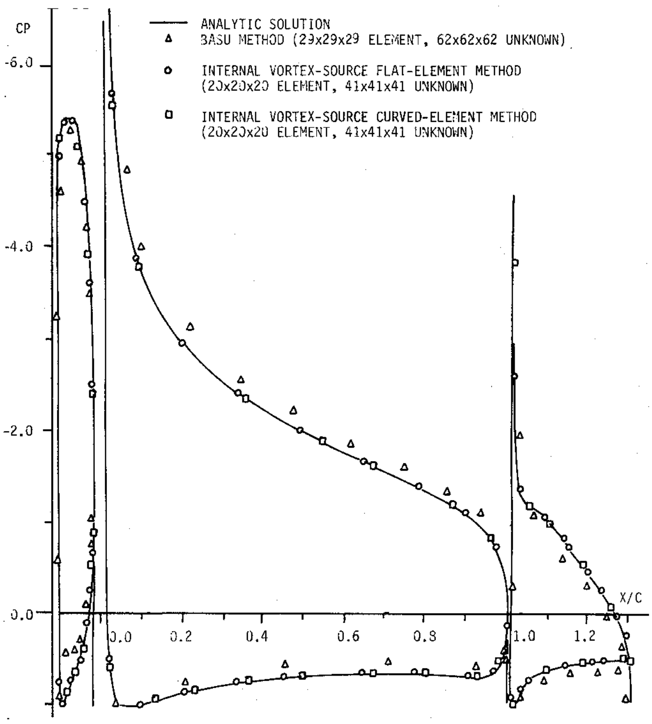

Fig. 3 Pressure distributions of internal singularity methods for three-element airfoil.

analytical solution, which can be determined by conformal transformation.[15] The internal integral equation method developed by Chen and Dalton [6] had the same numerical instability of the pressure distributions on the airfoil surface as that developed by Banyopadhyay et al.[7] The solutions of this numerical method are compared with those of the internal singularity method that was developed by Banyopadhyay et al.[7]

The modification of this method by using the curved element modeling of Hess[3] is applied to the multielement airfoils in order to investigate the effects of the curved elements on the distributions of the pressure and the strength of the singularity of this numerical method.

II. Analysis of Internal Singularity Method

This numerical method was developed and is shown in Fig. 1. The mean camber line is divided into N elements from the leading edge to the trailing edge (N unknown linear variations of source strengths are placed on the midpoint of each element, and $N+1$ unknown linear variations of vorticity strengths are placed on both ends of each element). Distributions of the vorticity and source strengths vary linearly across each element on the mean camberline inside the airfoil. These $(2N+1)$ unknowns are determined by satisfying the boundary condition of tangency of flow at $2N$ control points on the airfoil surface together with appropriate Kutta condition at the trailing edge. Corresponding to the midpoint of each element on the mean camber line, two points can be defined on the upper and the lower airfoil surfaces from the intersection of normals drawn from the mean camber line with the airfoil profile (Fig. 1). This identifies $2N$ control points on the airfoil surface where the tangency boundary conditions are to be satisfied. The Kutta condition is satisfied by equating the tangential velocities at the control points adjacent to the trailing edge on the upper and the lower airfoil surfaces.

In the methods that were developed in Refs. 6 and 7, a small gap was maintained between the leading edge and the element on the mean camber line closest to the leading edge of the airfoil in order to avoid infinite velocity near the leading edge. The size of the gap needed to be chosen carefully (see Refs. 6 and 7) or it would cause instability of the pressure distributions on the airfoil surface. This method does not have such disadvantage, i.e., there is no gap to be left between the leading edge and the element on the mean camberline closest to the leading edge. The effect of the higher-order approximation is investigated by a modification of this method with the application of curved element modeling of Hess.[3] (The airfoil surface and the the mean camberline can be represented by a number of curved elements, and the distributions of the vorticity and source strengths are parabolically varying across each curved element on the mean camber line.)

The conformal transformation of multiple circles onto multielement airfoils is a modification of the method developed by Suddhoo and Hall,[14] i.e., the general Kármán-Trefftz transformation is used, and there are no coordinate translation and rotation in the calculation.

III. Results and Discussions

The pressure distributions for three internal singularity methods on the two-element airfoil are shown in Fig. 2. The agreement between the analytic solution and the numerical solutions for both the internal linear-vortex-source flat element and curved element methods is excellent. The pressure distributions on the first element calculated by the method of Ref. 7 is a little greater than the analytic solution.

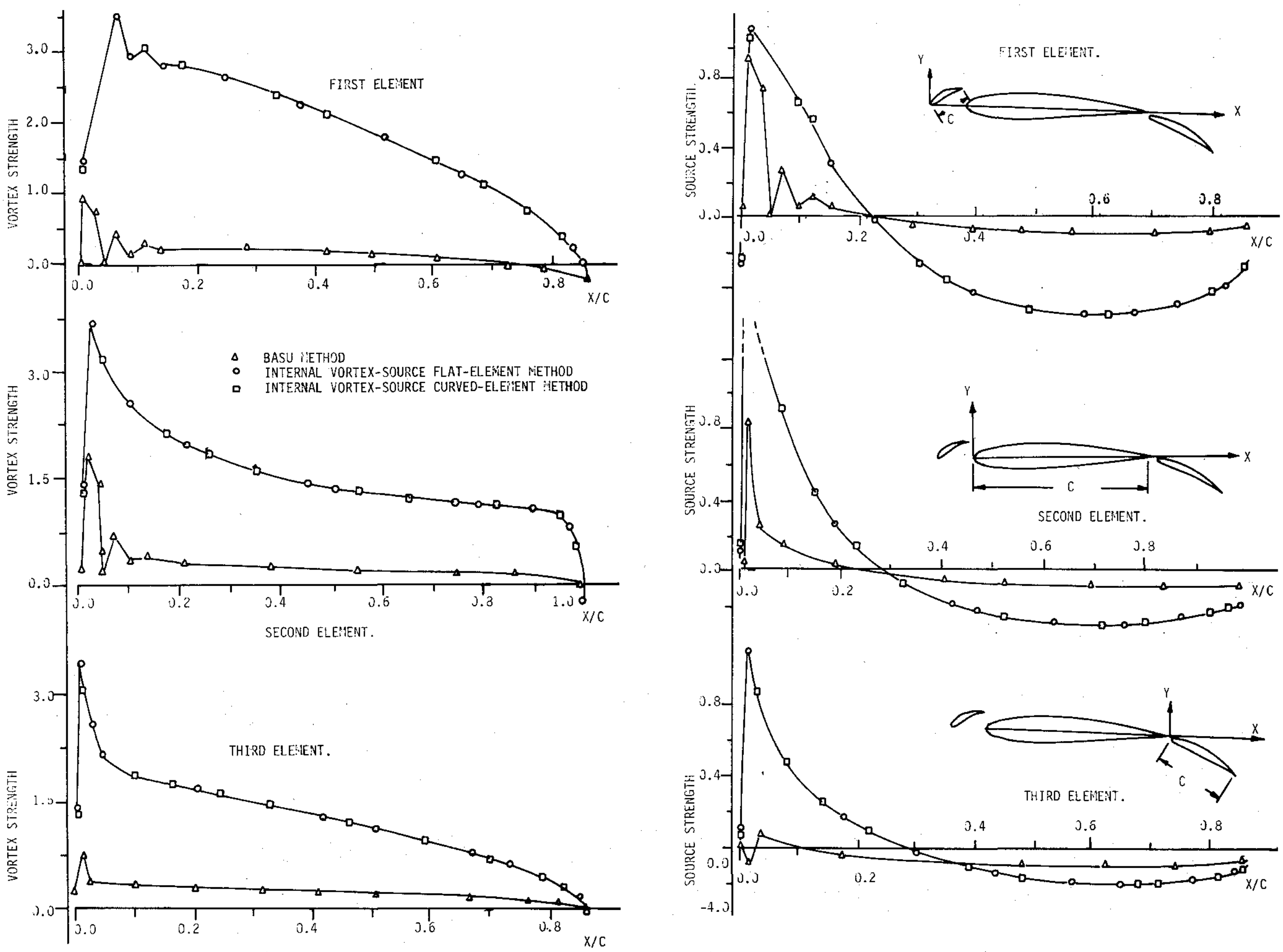

Fig. 4 Distributions of internal singularities of three-element airfoil.

The pressure distributions on the three-element airfoil are shown in Fig. 3. The results calculated by the method of Ref. 7 are very poor for the second and third airfoil elements when compared with the analytic solution. The agreement between the analytic solution and the solutions of both the internal linear-vortex-source flat element and curved element methods is good.

In order to investigate the discrepancy between the analytical solution and the numerical solution, which is calculated by the method developed by Ref. 7, the distributions of the source and vorticity strengths on the mean camber line of the three-element airfoil are shown in Fig. 4. The agreement of the distributions of the source and the vorticity strengths between the internal linear-vortex-source flat element and curved element methods is good. However, the distributions of the strengths of the singularities for the method of Ref. 7 are unstable in the region close to the nose of each airfoil element. The instability of the distributions of the singularities on the mean camber line of the three-element airfoil for the method of Ref. 7 causes the poor results of the pressure distributions that are shown in Fig. 3. It is suspected that due to the interferences of the source and the vorticity distributions on the mean camber line among the three airfoil elements, the criterion of the gap determination may need severe restriction for the method of Ref. 7.

IV. Conclusions

A flat element method based on linear distributions of vorticity and source on the mean camber line of the airfoil elements offers an approach to the solution of general potential flow problems. This method does not need any gap to be left between the leading edge and the element on the mean camber line closest to the leading edge to avoid the instability of the numerical solutions. In view of the good agreement and the accuracy of the pressure distributions obtained by the flat element and the curved element methods, the higher-order approximation with curved element of this numerical method may be unnecessary in the calculation.

References

[1]Hess, J. L. and Smith, A. M. O., "Calculation of Potential Flow about Arbitrary Bodies," *Progress in Aeronautical Science*, Vol. 8, Pergament Press, New York, 1967, pp. 1-138.

[2]Bristow, D. R., "Recent Improvements in Surface Singularity Methods for the Flow Field about Two-Dimensional Airfoils," AIAA Paper 77-641, June 1977.

[3]Hess, J. L., "Higher Order Numerical Solution of the Integral Equation for the Two-Dimensional Neumann Problem," *Computer Methods in Applied Mechanics and Engineering*, Vol. 2. No. 1, 1973, pp. 1–15.

[4]Hess, J. L., "The Use of Higher-Order Surface Singularity Distributions to Obtain Improved Potential Flow Solutions for Two-Dimensional Lifting Airfoils," *Computer Methods in Applied Mechanics and Engineering*, Vol. 5, 1975, pp. 11–35.

[5]Maskew, B. and Woodward, F. A., "Symmetrical Singularity Model for Lifting Potential Flow Analysis," *Journal of Aircraft*, Vol. 13, Sept. 1976, pp. 733–734.

[6]Chen, S. J. and Dalton, C., "Unsteady Lifting Case by Means of the Interior-Singularity Panel Method," *Journal of Aircraft*, Vol. 19, Aug. 1982, pp. 634–638.

[7]Banyopadhyay, G., Singh, B. K., and Basu, B. C., "Application of Internal Singularity Distribution Approach to Multi-Element

Airfoil Problem," *Aeronautical Journal*, May 1983, pp. 189–190.

[8]Chen, D. R. and Sheu, M. J., "Comarison of Numerical Solutions of Lower Order and Higher Order Integral Equation Methods for Two-dimensional Aerofoils," AIAA Paper 86-2591, Sept. 1986.

[9]Williams, B. R., "An Exact Test Case for the Plane Potential Flow about Two Adjacent Lifting Aerofoils," Royal Aeronautical Establishment R&M No. 3713, 1971.

[10]James, R. M., "Analytical Studies of Two-Element Airfoil Systems," Douglas Aircraft Company, Dept. HDC J5831, 1974.

[11]Garrick, T. E., "Potential Flow about Arbitrary Biplane Wing Section," NACA Rept. 542, 1936.

[12]Ives, D. C., "A Modern Look at Conformal Mapping Including Multiple Connected Region," *AIAA Journal*, Vol. 14, Aug. 1976, pp. 1006–1011.

[13]Halsey, N. D., "Potential Flow Analysis of Multielement Airfoils Using Conformal Mapping," *AIAA Journal*, Vol. 17, Dec. 1979, pp. 1281–1288.

[14]Suddhoo, A. and Hall, I. M., "Test Case for the Plane Potential Flow Past Multi-Element Aerofoils," *Aeronautical Journal*, Dec. 1985, pp. 403–414.

[15]Milne-Thomson, L. M., "Theoretical Hydrodynamics," Macmillan, New York, 1967.

Recommended Reading from the AIAA Progress in Astronautics and Aeronautics Series . . .

Spacecraft Dielectric Material Properties and Spacecraft Charging

Arthur R. Frederickson, David B. Cotts, James A. Wall and Frank L. Bouquet, editors

This book treats a confluence of the disciplines of spacecraft charging, polymer chemistry, and radiation effects to help satellite designers choose dielectrics, especially polymers, that avoid charging problems. It proposes promising conductive polymer candidates, and indicates by example and by reference to the literature how the conductivity and radiation hardness of dielectrics in general can be tested. The field of semi-insulating polymers is beginning to blossom and provides most of the current information. The book surveys a great deal of literature on existing and potential polymers proposed for noncharging spacecraft applications. Some of the difficulties of accelerated testing are discussed, and suggestions for their resolution are made. The discussion includes extensive reference to the literature on conductivity measurements.

TO ORDER: Write AIAA Order Department,
370 L'Enfant Promenade, S.W., Washington, DC 20024
Please include postage and handling fee of $4.50 with all orders.
California and D.C. residents must add 6% sales tax. All orders under $50.00 must be prepaid. All foreign orders must be prepaid. Please allow 4-6 weeks for delivery. Prices are subject to change without notice.

1986 96 pp., illus. Hardback
ISBN 0-930403-17-7
AIAA Members $26.95
Nonmembers $34.95
Order Number V-107